Energy and Magnetic Fields

Recall that the **energy stored** in an **electro**static system is:

$$W_e = \frac{1}{2}\iiint_V \rho_v(\bar{r})\, V(\bar{r})\, dv$$

or equivalently:

$$W_e = \frac{1}{2}\iiint_V \mathbf{D}(\bar{r}) \cdot \mathbf{E}(\bar{r})\, dv$$

This led to the expression relating energy and **capacitance**:

$$W_e = \frac{1}{2} C V^2$$

We can similarly ask the question, how much **energy** is stored in a **magneto**static system?

Precisely the amount of **work** required to establish the **current density** $\mathbf{J}(\bar{r})$!

We find that the expressions for this work/energy are **analogous** to that of electrostatics. For example, we find that:

$$W_m = \frac{1}{2}\iiint_V \mathbf{J}(\bar{r}) \cdot \mathbf{A}(\bar{r})\, dv$$

Therefore, we **again** find that energy stored is equal to the integration of the "product" of the **sources** (e.g., ρ_v or $\mathbf{J}$) and the **potential** function (e.g., V or $\mathbf{A}$).

Likewise, this energy can be expressed in terms of the two magnetic **fields**:

$$W_m = \frac{1}{2}\iiint_V \mathbf{B}(\bar{r}) \cdot \mathbf{H}(\bar{r})\, dv$$

Therefore, we again find that energy stored is equal to the integration of the dot product of the **flux density** (e.g., $\mathbf{D}$ or $\mathbf{B}$) and the other **field** (e.g., $\mathbf{E}$ or $\mathbf{H}$).

We likewise find that this energy can be directly expressed for the energy stored by an **inductor**:

$$W_m = \frac{1}{2} L I^2$$

Look **familiar** ?